# PYTHAGORAS in LONDON

## Guia de treball

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**Material elaborat per www.mat3.cat**
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## AVATAR

An **Avatar** is a personal icon in a virtual context.

We will develop the capacity to program and control the avatar's movements and we will learn how to use a powerful mathematics tool which is necessary to program an avatar.

## *A. SPECIAL EQUATIONS*

**A.1.** Calculate without calculator and memorize the following results:

**a)** $2^2$, $3^2$, $4^2$, $5^2$, $6^2$, $7^2$, $8^2$, $9^2$, $10^2$, $11^2$, $12^2$, $13^2$ i $14^2$

**b)** $\sqrt{1}$ , $\sqrt{4}$ , $\sqrt{9}$ , $\sqrt{16}$ , $\sqrt{25}$ , $\sqrt{36}$ , $\sqrt{49}$ , $\sqrt{64}$ , $\sqrt{81}$ …

**A.2.** Solve the following quadratic equations:

**a)** $x^2 = 64$

**b)** $x^2 - 6 = 30$

**c)** $10 - x^2 = 9$

**d)** $x^2 = 2$

**e)** $7^2 + x^2 = 2$

**f)** $12 = 1{,}3 + x^2$

**g)** $11^2 + x^2 = 13^2$

## *B. A LITTLE BIT OF VOCABULARY AND BASIC CONCEPTS*

**B.1.**We need to learn specific vocabulary. Write the definitions of the following words and draw an illustrative picture.

**a)** Equilateral triangle

**b)** Isosceles triangle

**c)** Escalene triangle

**d)** Obtuse triangle

**e)** Rectangle triangle or right triangle

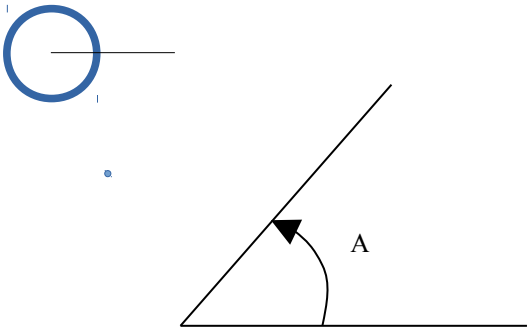
**f)** Acutangle triangle

**g)** Hypotenuse

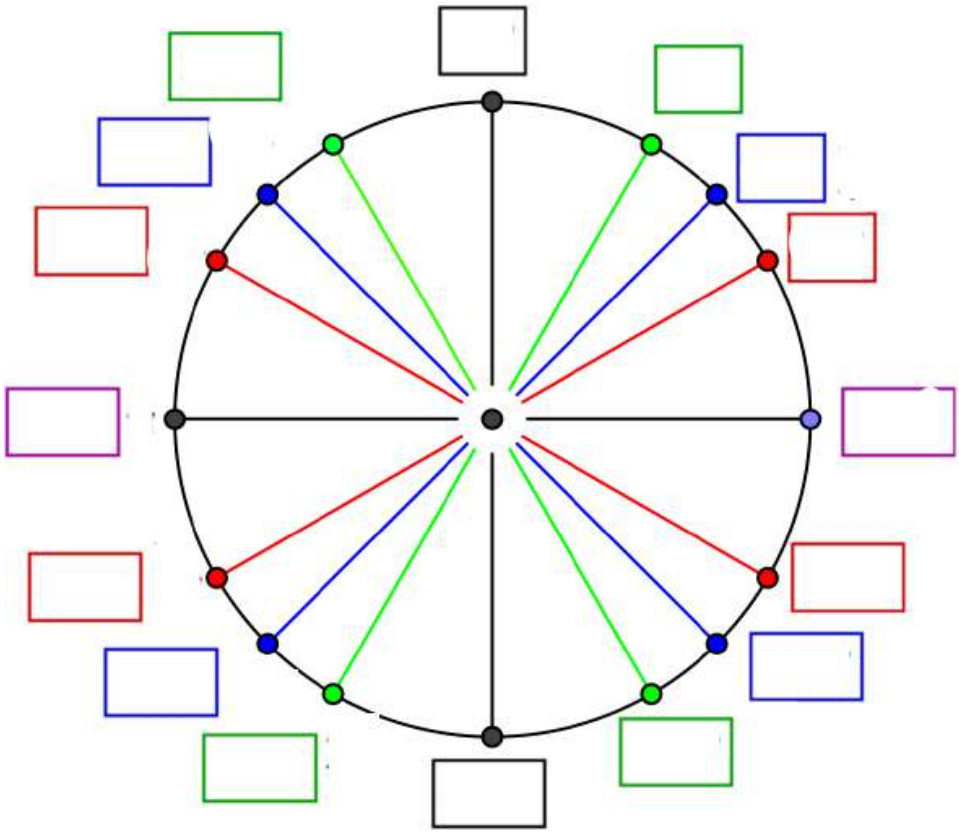
**h)** Cathetus

**B.2.** Remember that we measure the angles from the horizontal side to the other one in the counterclockwise.

Remember also that one turn is 360º (degrees).




Write the measure of each angle:



**B.3.** Try to do the same figure in the Geogebra and add the exercise in the Moodle. (surname_B3_angles_2X.ggb)

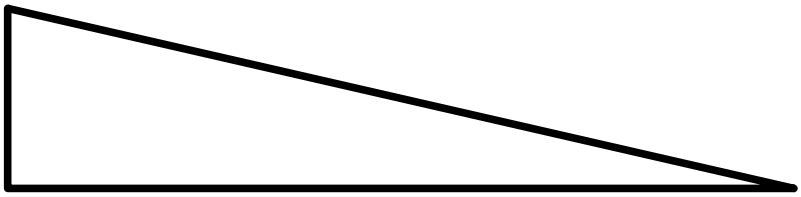
**B.4.** Draw the following angles 37º, 152º, 205º, 294º, -25º and 422º using a protractor.

**B.5.** We use small letters to name sides of a triangle and capital letters to name angles. Furthermore, if one side is named **a**, the **opposite** angle will be **A**, if one side is named **b**, the **opposite** angle will be **B** and finally if one side is named **c**, the **opposite** angle will be **C**.

On the other hand, sides b and c are **contiguous** to A.

Normally, if the triangle is rectangle we will name A to the right angle.

Write de definition of **opposite** and **contiguous** and name sides and angles to the following triangle:



**B.6.** Draw a triangle in a paper. Cut the three angles and join them.

**a)** What is the sum of the three angles?

**b)** What about the addition of your colleagues?

**B.7.** What is the sum of the four angles from a square? Why?

**B.8.** What is the value of one angle from a regular pentagon?

**B.9.** What is the value of one angle from a regular hexagon?

## *C. FIRST STEPS PROGRAMMING*

**C.1.**Write the translation in catalan of the following words.

*Move, step, turn, degree, clear, pen down, pen up.*

**C.2.** Playing with avatars (in this case, the avatar will be a robot because we will move in a real environment)

One student will be a robot and the other one will be the instructor (person who gives instructions). The "robot" goes out of the classroom. The instructor can only use the words: *Move, step, turn and degree.*

The teacher will give to the instructor a message and the instructor has to achive to move the robot.

All the students have to write the instructions and have to guess the robot's movements.

**C.3.** The teacher will give a drawing to the instructor (who will show it to the others except for the robot or avatar). The instructor will have to achieve the robot draw the picture on the blackboard using the words

*Move__step, turn__degree, clear, pen down, pen up.*

**C.4.** Now, everybody will be an avatar or an instructor.

**a)** Draw a picture in your notebook using squared paper. The picture has to have only segments and right angles.

**b)** Write the instructions to draw it in a paper.

**c)** Change the instructions with a colleague.

**d)** Draw the picture of your colleague following the instructions.

**e)** Compare the drawings. Did you get it?

**C.5.** Write the instructions for an avatar who walks one millimetre by one steep in order to make the following figures:

**a)** a square of 100 mm side

**b)** an equilateral triangle of 100 mm side

**c)** a pentagon of 50 mm side

**d)** a hexagon of 50 mm side

**C.6.** Open Scratch, insert an avatar (costume)  and draw the pictures using the following instructions:


**a)** a square of 100 mm side

**b)** a equilateral triangle of 100 mm side

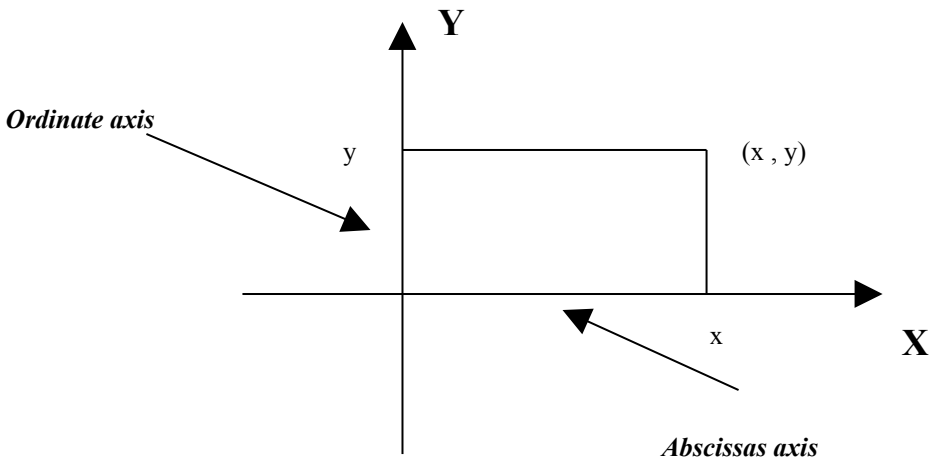
**c)** a pentagon of 50 mm side

**d)** a hexagon of 50 mm side

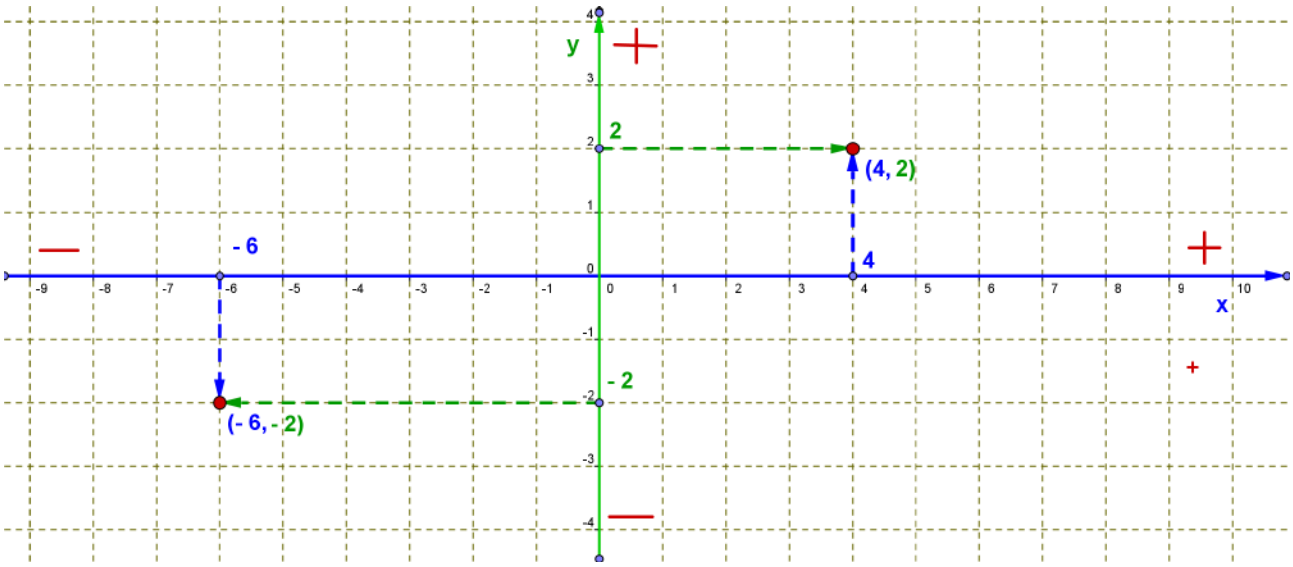
**C.7.** Program your avatar Scratch to draw your figure made in exercise C.4.

## *D. COORDINATE*

When we want to orientate an object in some computer systems, robotic systems, etc. we need the **coordinates** in order to locate it. Look at this:




So, if we want to locate the point (4,2), we should go 4 units right and 2 units up. Another example, if you want to locate the point (-6,-2), we should go 6 units left and 2 units down.




**D.1.** Draw a coordinate axis in your notebook and locate the following points: (2, 4); (3,-6); (-2, 5); (-5, -3); (6,2); (-2, 3); (4, -6); (-1,-1). Check the solution in Geogebra.

**D.2.** Draw a coordinate axis in your notebook and locate the following points: (-6, -1), (-5, -3), (4, -3), (5, -1), (-1, -1), (1, 2), (-1, 3) and (-1, 0). Join the points. What is the result?

**D.3.** Draw a picture using lines and points in a coordinate axis, it's necessary to add at least a negative coordinate. Order the points and write them in your notebook. Change your points with a colleague and try to guess his or her drawing.

## *E. GEOGEBRA VERSUS SCRATCH*

GeoGebra and Scratch are two different programmes. Geogebra is a Maths' programme where we can understand and learn a lot of Maths. Scratch is more recreational where we can program avatars. However, to use Scratch it's necessary to learn some mathematical tools.

**E.1.** Think about drawing a triangle in GeoGebra and research how can you calculate an angle.

**a)** Explain the instructions to draw a triangle in GeoGebra knowing the sides.

**b)** Explain the instructions to calculate the value of an angle in GeoGebra.

**c)** Draw the triangle of sides a = 10, b = 6 and c = 8 units and calculate the angles A,B and C.

**E.2.** Draw the following triangles in GeoGebra and calculate all their angles. Identify if the triangle is obtuse, rectangle or acutangle.

**a)** a = 9, b = 9 and c = 2

**b)** a = 7, b = 4 and c = 10

**c)** a = 9, b = 4 and c = 2

**d)** a = 5, b = 4 and c = 3

**E.3.** We know that a triangle is rectangle and the cathetus are b = 6 and c = 8. Draw the triangle in GeoGebra and measure the value of the hypotenuse and the others angles.

**E.4.** Draw a triangle in Scratch with sides a = 100, b = 70 and c = 50. Is it possible? Which kind of difficulties do you have? Write them in your notebook.

**E.5.** If we want to draw a triangle in Scratch, it will be easier knowing sides and angles. So, using the results of the exercise E.2. draw the following triangles in Scratch.

Be careful: we assume that GeoGebra uses a centimetre as unit and Scratch uses a millimetre as unit)

**a)** a = 90, b = 90 and c = 20

**b)** a = 70, b = 40 and c = 100

**c)** a = 90, b = 40 and c = 20

**d)** a = 50, b = 40 and c = 30

**E.6.** Draw a right triangle in Scratch with cathetus b= 60 and c = 80. Use the results of exercise

Conclusions:

If you want to program in Scratch, you will need powerful mathematical tools to calculate sides and angles. GeoGebra could be a good option. However you can learn a new tool. Come on!

## *F. PYTHAGORAS*

**F.1. Pythagoras** is one of the most important mathematician of history. In order to know aspects about his personality, we will work in groups using the puzzle technique. Each of you will be an expert in the following aspects:

- General information about Pythagoras life and his sect.
- Anecdotes about Pythagoras life and his famous sentences.
- The importance of the Pythagorean sect in the history including its final.
- Some numerical properties studied by the Pythagorean (excluded Pythagorean theorem)

Share your information and write an explanatory text in your notebook. Emphasise the historical personality of Pythagoras.

**F.2.** The property of numbers is very important for Pythagoreans. A **pythagorean triple** is three numbers (a,b,c) with the property $a^2 = b^2 + c^2$.

**a)** Verify that (5,4,3) is a pythagorean triple.

**b)** Look for at least two pythagorean triples.

**c)** Write in your notebook all the pythagorean triple searched by your colleagues.

**F.3.** If you want to find a pythagorean triple, you can choose two numbers and calculate the third one by calculator. Observe: numbers can be decimals.

Calculate the third value of the following pythagorean triple (write **four** decimals). Write in your notebook all the calculations.

**a)** knowing b = 10 and c = 14, calculate the value of a

**b)** knowing b = 4,5 and c = 6,8, calculate the value of a

**c)** knowing a = 12 and b = 5, calculate the value of c

**d)** knowing a = 7,3 and b = 4,6, calculate the value of c

**e)** knowing a = 11 and c = 8, calculate the value of b

**f)** knowing a = 9,2 and c = 5,3, calculate the value of b

**F.4.** Choose three pythagorean triples and:

**a)** Draw a triangle in GeoGebra for each pythagorean triple. (a, b, and c will be the sides).

**b)** Classify each triangle as obtuse, rectangle or acutangle.

**c)** Do you recognise any property?

**F.5.** In fact, it is not by chance. It will be true in the opposite sense? Pay attention!

**a)** Draw whatever **rectangle triangle** in GeoGebra.

**b)** Measure the three sides and name **a** as hypotenuse and **b** and **c** as cathetus. (write at least two decimals).

**c)** Is (a,b,c) a pythagorean triple?

**d)** How many of you have a pythagorean triple, ie, $a^2 = b^2 + c^2$ ?

**F.6.** Now, we want to know if the property of the pythagorean triple is true for all kind of triangles.

**a)** Draw any **triangle** in GeoGebra.

**b)** Measure the three sides and name **a** as hypotenuse and **b** and **c** as cathetus. (write at least two decimals).

**c)** Is (a,b,c) a pythagorean triple?

**d)** How many of you have a pythagorean triple, ie, $a^2 = b^2 + c^2$ ?

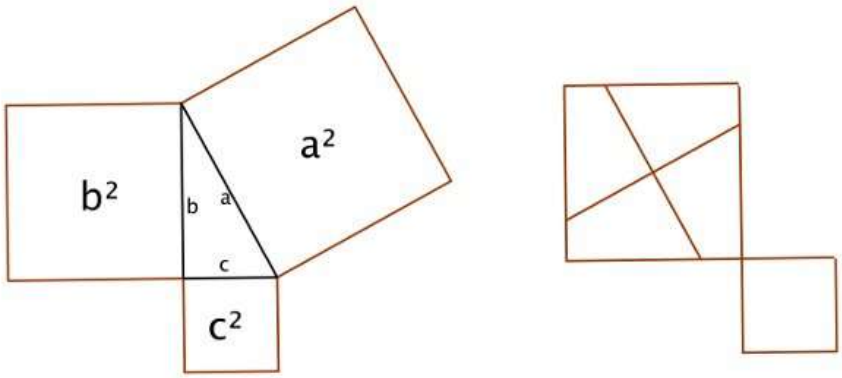
## *G. PYTHAGORAS' THEOREM*

If we draw a triangle using a (a,b,c) pythagorean triple, it will be a right triangle. Furthermore, if we draw a right triangle, we will obtain a pythagorean triple (a,b,c,) where **a** is the hypotenuse and **b** and **c** cathetus.

However, why?

**G.1.** Open the file **pythagoras.ggb** (it is in Moodle). Observe the figure and manipulate it. Explain in your notebook the reason why pythagorean triples are always sides of a right triangle.

**G.2.** Now, using paper. The teacher will give you a puzzle which is similar to the GeoGebra one. Try to do the same, cutting the paper and pasting it in your notebook.

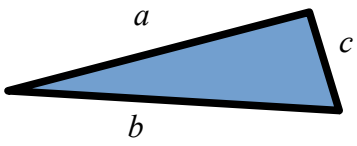


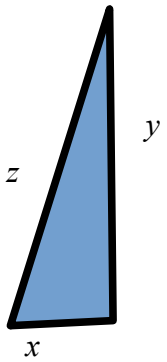

**G.3.** Look at the following video https://www.youtube.com/watch?v=gHJNMiSFuAM and write the definition of **theorem** in your notebook. (Remember the rules of a definition)

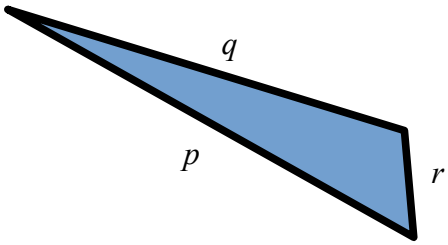
**G.4.** Think about the formulation of Pythagoras' theorem. Start to write "Pythagoras' theorem says that…...". Share your definition with your colleagues and write the best definition.

**G.5.** Look for demonstrations of Pythagoras' theorem by Internet. Choose one demonstration and explain it in your notebook. Then, present your option to your colleagues.

**G.6.** Normally we use **A** to name the right angle and **a** to name hypotenuse. However, it could be different. Write the Pythagoras' theorem in the following pictures and share it with your colleagues.

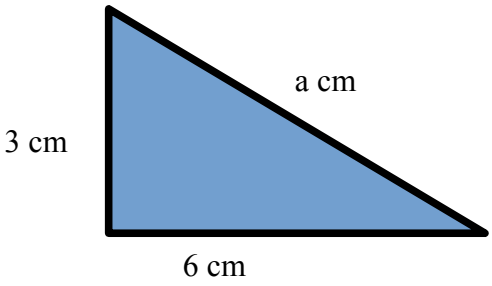


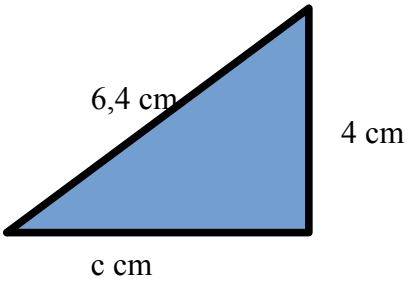




**G.7.** Before continuing, we need to practice a little bit with some examples. The following triangles are right-angled, look for the missing side using Pythagoras' theorem and a calculator.

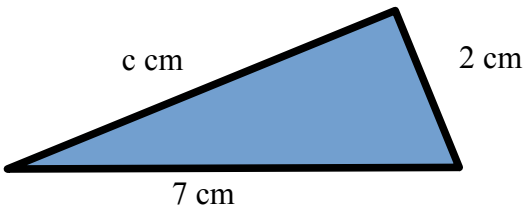
**a)**




b)

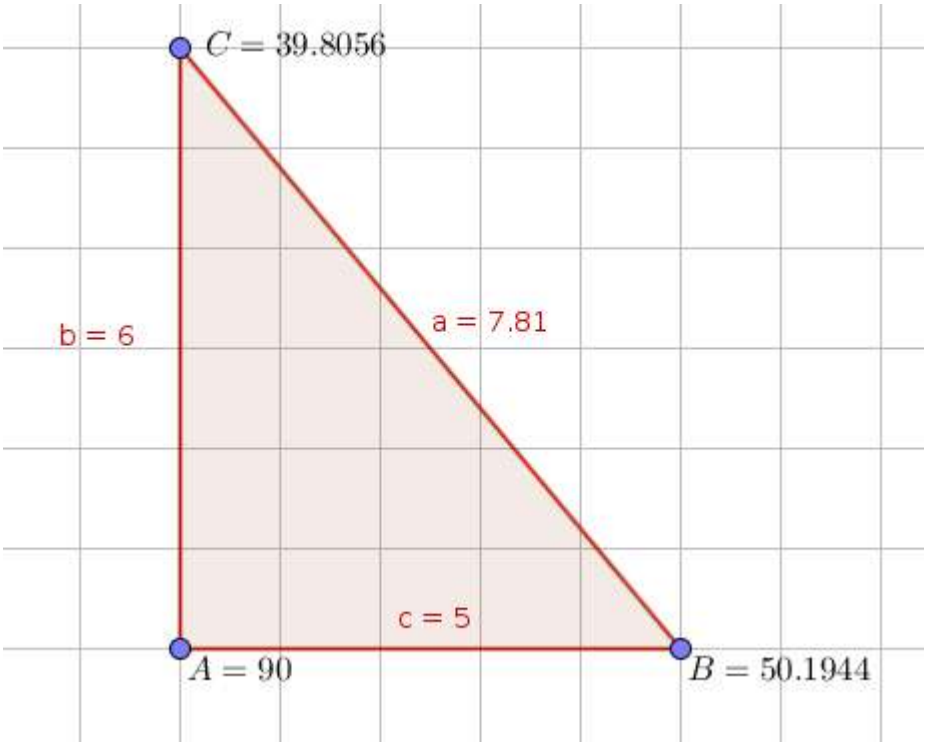



c)



## *H. NOW WE ARE GOING TO STUDY ANGLES: THE TANGENT*

Observe the following right triangle, its sides and angles.




In a right angled triangle, the **tangent** of an angle is the length of the **opposite side** divided by the length of the **adjacent side**. It's calculated by the calculator using the key **tan** $\tan(B)=\frac{b}{c}$ (also **tg** $tg(B)=\frac{b}{c}$ )

Prove it:



The most important thing is the opposite key (shift), so we can find the angle if we know the opposite side and the adjacent side of an angle in a right triangle.

Mathematically we write: $B=\arctan(\frac{6}{5})=50.194428891^o$

and it means 50,194428891 is the angle (arc) which tangent is $\frac{6}{5}$ .

**H.1.** Open the file **triangle.ggb** (in Moodle). Move points B and C and observe that the angle A is always of 90º. Using the length of sides and the calculator, calculate five angles. Write the value of the opposite side and the adjacent side and the value of the angle with 6 decimal.

Example: If *b = 6 and c= 5 then the angle B will be:* $B=arctg(\frac{6}{5})=50.194428891^o$

**H.2.** If we want to calculate angle C, do you know how we can do it?

**a)** Write in your notebook how to find the value of the angle C.

**b)** Draw five right triangles and calculate the angle C in any case. (Remember to use 6 decimal)

Example : If *b = 6 i c = 5 then the angle C will be:* $C=arctg(\frac{5}{6})=39.80557109^o$

**H.3.** Which is the relation between the angles B and C (of the same right triangle)?

## *I. AVATARS*

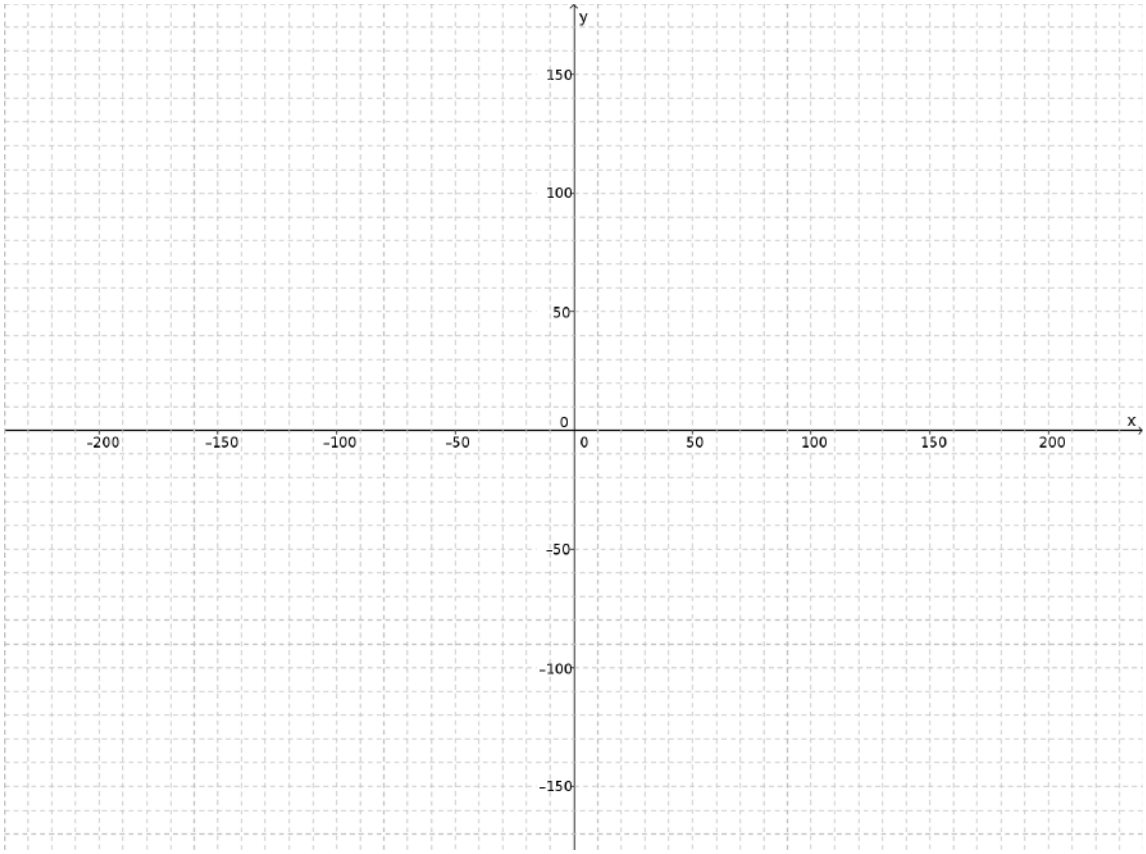
One of the most important goals in robots' programming is to deactivate bombs in a land after a war. Another one is to find people alive after a natural disaster, like an earthquake. Other goals are to explore the moon or other planets or simply to clean our home.

**I.1.** In a land-mine we have marked coordinates. A robot with a magnetic sensor has detected some mines in the following coordinates:

A = (70, 40), B = (-110, 120), C = (210, 80), D = (-100, -130), E = (50, -140), F = (210, 30),

G = (20,130), H = (90, -90), I = (-40, -30).

**a)** Place the points in the following coordinate axis:




**b)** Download **mines1.jpg** (in Moodle) in your computer and the correct previous activity.

**c)** Programme by Scratch an avatar which deactivates all mines. Follow the next:

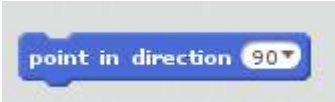
- Open the Scratch
- Insert the image *mines1.jpg* as a "*backdrop"*
- Put your own avatar and decrease the size.
- Programme the following movement in order to put your avatar in the centre point looking at the right side and clean the screen. It is activated by pressing key **i** .


Observation: 'Avatar looks at the right side' means 'point in direction 90º'. It's a rule only for Scratch

- Now, programme the avatar in order to go to all the mines and mark the path (*pen down)*. Firstly, the avatar has to go horizontally, X axis, and then vertically, Y axis. Finally, avatar has to come back diagonally, by the hypotenuse of the right triangle. You can only use the instructions: *Move__step, turn__degree, clear, pen down, pen up.*

  (If you want, you can also use the instruction s*ay* ____ *for* __ *secs*, for example s*ay MINE for 2 secs*)


Be careful, add an orientation of the avatar to the right side in order to start again with another triangle (or mine):




- Pay attention, you have to calculate all the hypotenuses and all the angles. Write all the calculus and a picture in your notebook carefully.

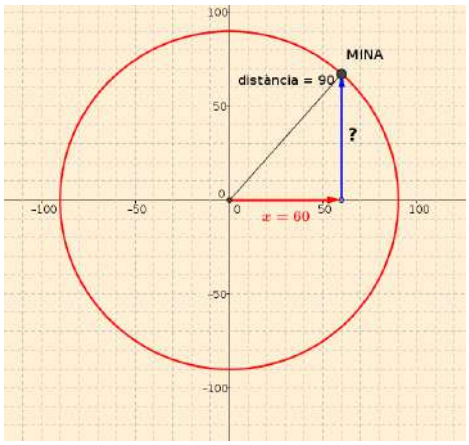
**I.2.** In another land-mine, a robot has two sensors, one is the distance of the centre of the coordinate axis ans the other is the length of the X axis and the sing of the Y axis (positive >0 or negative <0)

This is an example:

a distance to the centre of 90 units

the x coordinate is 60 units

and the y coordinate is positive (>0).




Programme by Scratch your avatar deactivated the following mines:

| **Distance to the centre** | ***x coordinate*** | ***Y sign*** |
|---|---|---|
| 110 | 80 | $y > 0$ |
| 40 | - 30 | $y > 0$ |
| 75 | 50 | $y < 0$ |
| 64 | - 45 | $y < 0$ |
| 69 | - 55 | $y > 0$ |
| 220 | -150 | $y > 0$ |
| 195 | - 80 | $y < 0$ |
| 43 | 32 | $y < 0$ |

Following the next steeps:

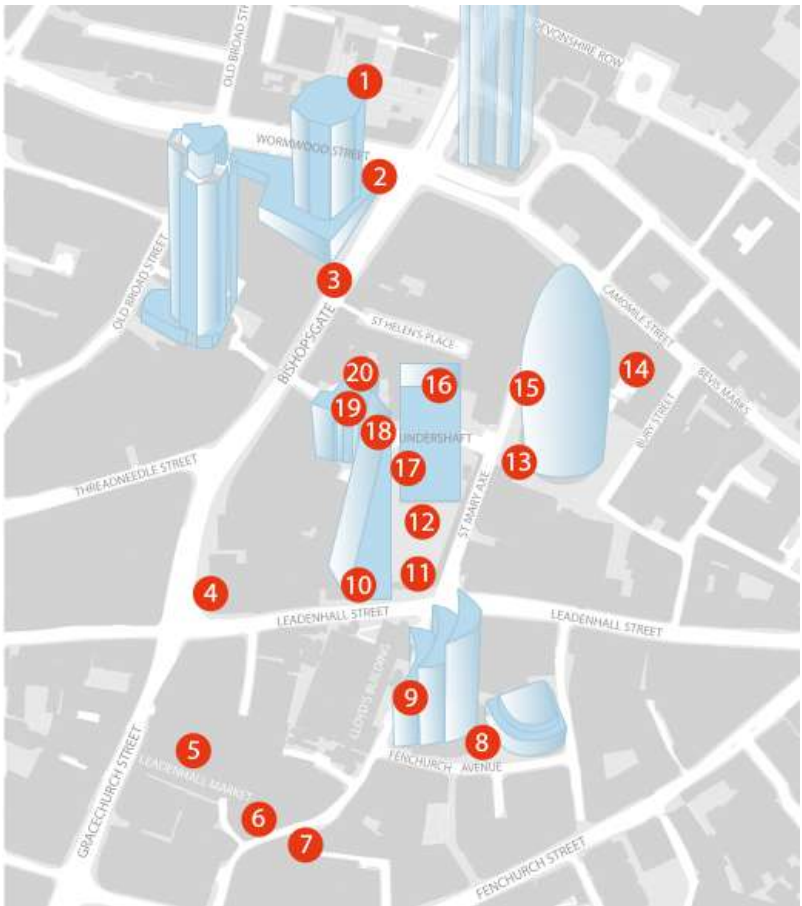
- Add the file **mines2.jpg** as a *backdrop* (in Moodle)
- For each mine, calculate the missing side and the necessary angles
- Firstly, avatar has to go horizontally, X axis, and then vertically, Y axis. Finally, avatar has to come back diagonally, by the hypotenuse of the right triangle. If you want, you can add sound when the avatar deactivates the mine.

## *J. PYTHAGORAS IN LONDON*


Look at the video https://youtu.be/2IJ65wlcw0U It is the project SCULPTURE IN THE CITY from 28 June 2016 to May 2017.

Our goal will be to find sculptures in the city of London and discover their impact in your perception.

***J.1.*** First of all, you have to know the distance between the sculptures in order to plan your pathway. Look at the map and find the distance between the sculptures.

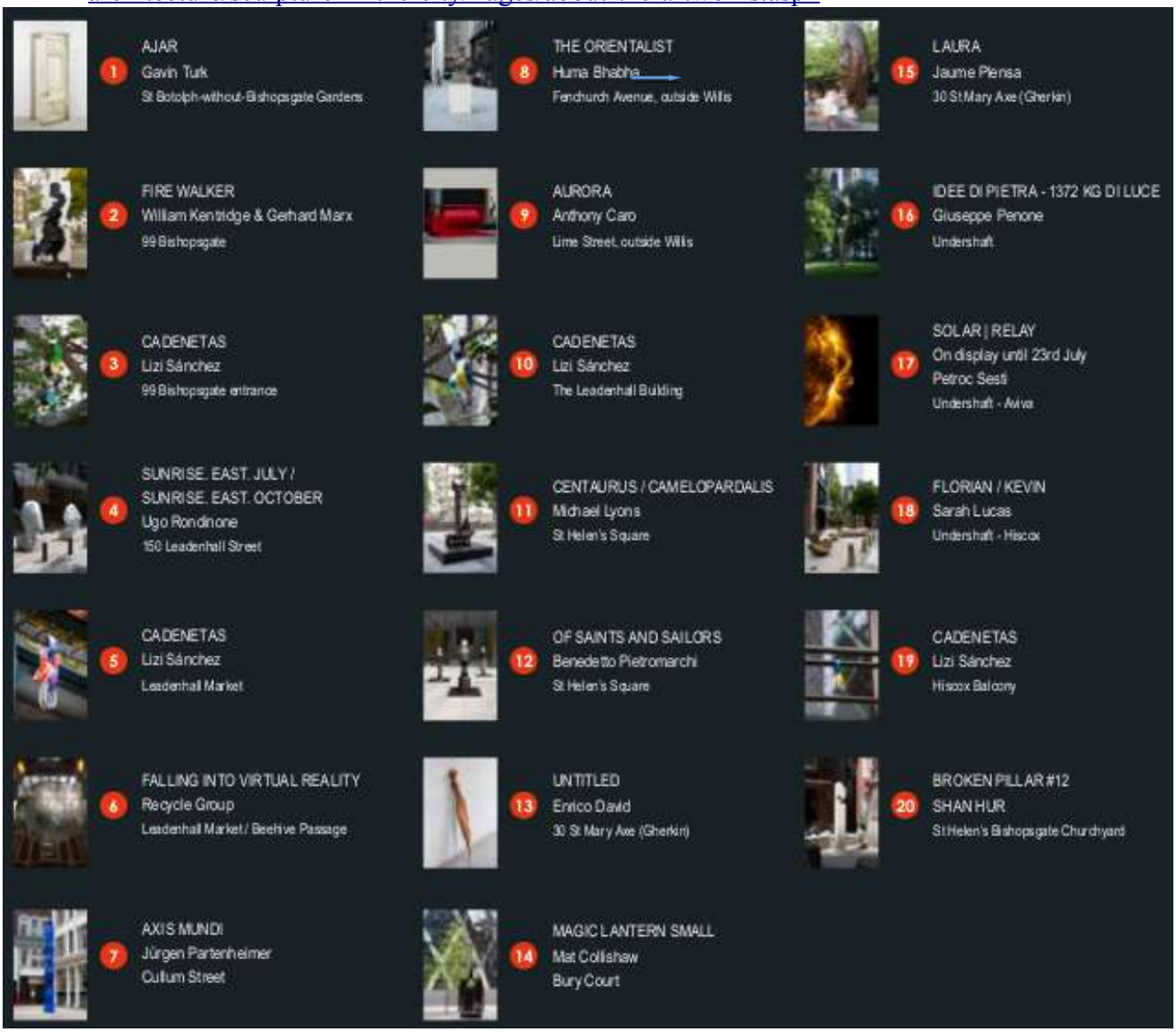



Reference: 100m is the distance is between the sculpture 4 "Sunrise. East. July/Sunrise. East. October" of Ugo Rondinone in 150 Leadenhall Street and the sculpture 10 "Cadenetas" of Lizi Sánchez in The Leadenhall Building.

**a)** Draw in GeoGebra points (sculptures) and write the distance between them drawing lines. The result will be a **graph.**

**b)** Is there any right triangle?

**J.2.** All the Sculptures are: https://www.cityoflondon.gov.uk/things-to-do/visit-the-city/art-architecture/sculpture-in-the-city/Pages/about-the-artworks.aspx



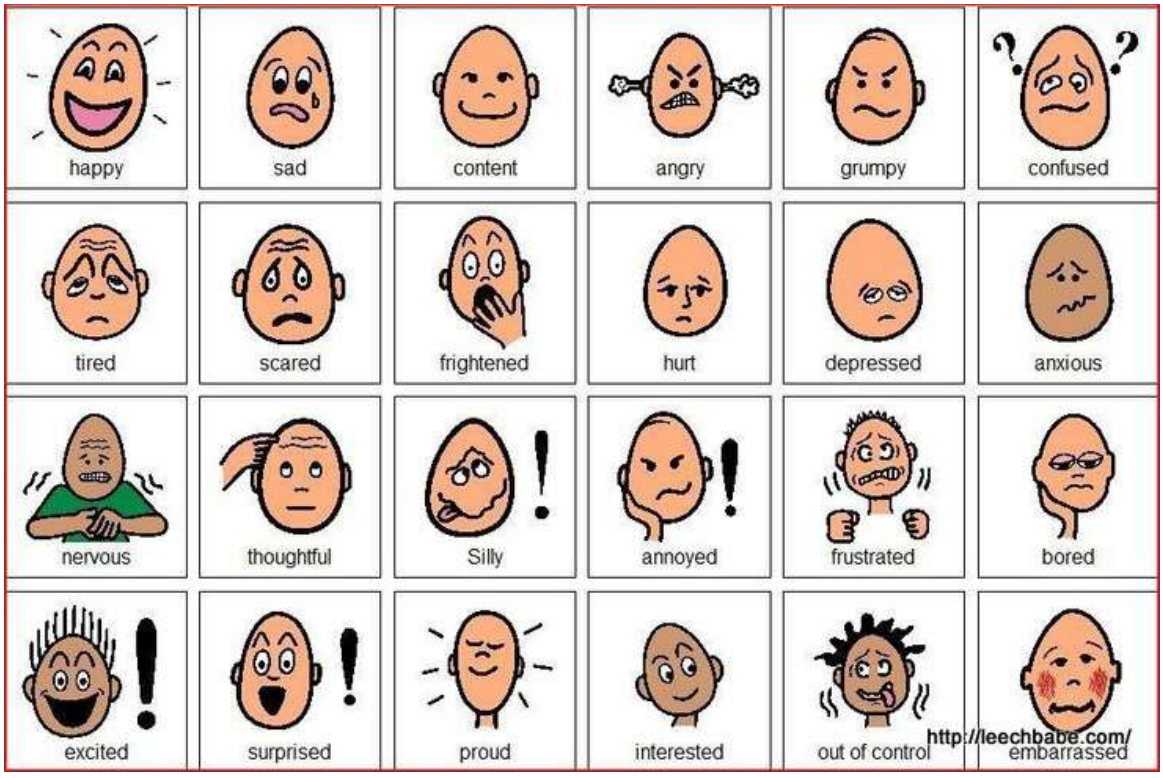
**J.3.** Draw in the same picture in GeoGebra the streets using a different coloured line. Write the distance in each line (using Object Properties… → Show level → **value**)

**J.4.** Define your path in order to find as many sculptures as possible.

For example, 4-10-11-12 is the pathway from the sculpture 4 to 10, from 10 to 11 and from 11 to 12.

**J.5.** Write the distance of your pathway and save the image in your mobile. You will need it in London.

**J.6.** When you are in front of the sculpture, watch it carefully and write what makes you feel (choose one of the feelings from the chart)




**a)** Write the title and the artist

**b)** Take a photo and add it in your notebook

**c)** Write at least 3 feelings related to the sculpture

**d)** Then, think of a sentence that could describe it

Example: 'Ajar' | Gavin Turk | 2011

feeling: happy

sentence: Open the door to my dreams